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LETTER TO THE EDITOR

Nonlinear susceptibility in the spin glass

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Abstract. Nonlinear susceptibilities in the SK model of the spin glass are studied in terms of a Parisi symmetry breaking scheme. It is shown that the nonlinear susceptibility χ_2 for an AC field is negatively divergent in the paramagnetic phase near the Almeida–Thouless line, and the χ_2 in the spin glass phase changes according to the frequency scale of the AC field.

It has been shown that the nonlinear susceptibility χ_2 has a negatively divergent behaviour at the spin glass transition temperature T_{g0} in the zero external field both theoretically (Katsura 1976, Suzuki 1977) and experimentally (Chikazawa *et al* 1979). In the non-zero static external field the spin glass phase is separated at the Almeida–Thouless (AT) line $T_g(H)$ (de Almeida and Thouless 1978) from the paramagnetic phase. In this letter it is shown that in the non-zero static external field, the χ_2 for the infinitesimal field with the same time scale as the static field has no singular behaviour, but the χ_2 for the infinitesimal field with a shorter time scale than the static field has a singular behaviour in the neighbourhood of the AT line in terms of a Parisi symmetry breaking scheme (Parisi 1980).

We follow the formulation by Parisi (1980) and the details are given in his paper. We consider the SK model (Sherrington and Kirkpatrick 1975) in small static and very small AC fields near $T_g(H)$. In this scheme the AC field is considered as the replica symmetry breaking in the correlation between the external fields with the different replica index $H_\alpha H_\beta$. In this system we have the free energy per spin

$$F(T)=\max_{\{Q\}} F(Q)$$

$$F(Q)=-\lim_{n\to 0}\frac{1}{2n}\left(\tau\,\mathrm{Tr}\,(Q^2)+\tfrac{1}{3}\mathrm{Tr}\,(Q^3)+\tfrac{1}{6}\sum_{\alpha,\beta}(Q_{\alpha\beta})^4+\sum_{\alpha,\beta}Q_{\alpha\beta}H_\alpha H_\beta\right)$$
$$+F(Q)|_{Q=0} \qquad (1)$$

where $\tau\equiv 1-T$ and $(Q_{\alpha\beta})$ is the matrix of the order parameter given by Parisi (1980) and we consider explicitly the external field with a replica index. After the Parisi parametrisation $H_\alpha H_\beta \to H^2(x)$, we have

$$H^2(x)=h^2 \qquad x<x_1$$
$$=h^2+h_1^2 \qquad x>x_1 \qquad (2)$$

where h is considered as a static field and h_1 as an AC field with the frequency scale x_1 and we assume $1\gg h^2\gg h_1^2$.

Calculating $\mathrm{d}F/\mathrm{d}Q_{\alpha\beta}=0$, we have

$$2q(x)(\tau-\bar{q})+\tfrac{2}{3}q^3(x)=\int_0^x (q(x)-q(x'))^2\,\mathrm{d}x'-h^2-h_1^2\theta(x-x_1);$$
$$\bar{q}\equiv\int_0^1 q(x)\,\mathrm{d}x \tag{3}$$

after performing the Parisi parametrisation.

First we consider the solution in the paramagnetic phase. We have a solution:

$$\begin{aligned} q(x)&=q_0 && x<x_1\\ &=q_0+\delta q && x>x_1 \end{aligned} \tag{4}$$

where q_0 and δq are determined by equations (3) and (4) and we have at $\tau\gg 2^{1/2}h$

$$q_0\simeq +\tfrac{1}{3}\tau^2+(h^2+h_1^2)/2\tau+[1+(1/2\tau)(\tfrac{4}{3}\tau^2-h^2/\tau)]x_1\delta q \tag{5}$$

$$\delta q\simeq h_1^2/(h^2/\tau-\tfrac{4}{3}\tau^2). \tag{6}$$

We define the time scale dependent susceptibility following the discussion of Hertz (1983):

$$G(x)\equiv\left(1-\bar{q}-\int_0^x (q(x)-q(x'))\,\mathrm{d}x'\right)\Big/T. \tag{7}$$

From this we have

$$\begin{aligned} G(x<x_1)&\simeq 1-\tau^2/3-(h^2+h_1^2(1-x_1))/2\tau\\ G(x>x_1)&\simeq G(x<x_1)-x_1h_1^2/(h^2/\tau-4\tau^2/3). \end{aligned} \tag{8}$$

As the AT line is given by $h^2=\tfrac{4}{3}\tau^3$, the $\chi_2(x>x_1)\equiv \mathrm{d}^2G(x>x_1)/\mathrm{d}h_1^2$ is negatively divergent at the AT line.

Next we consider the solutions in the spin glass phase. Differentiating equation (3) with respect to x, we have

$$2q'(x)\left(\tau-\bar{q}+q^2(x)-\int_0^x (q(x)-q(x'))\,\mathrm{d}x'\right)=-h_1^2\delta(x-x_1). \tag{9}$$

Differentiating equation (9) with respect to x, we have

$$q'(x)(2q(x)-x)=0 \tag{10}$$

where we have assumed that $x\neq x_1$ and $q'(x)\neq 0$.

We have three cases according to the value of the frequency scale x_1 of the AC field: case 1 is $x_1>2\tau$, case 2 is $2(\tfrac{3}{4}h^2)^{1/3}>x_1$, and case 3 is $2\tau>x_1>2(\tfrac{3}{4}h^2)^{1/3}$.

First we consider case 1. Here we have the next solution

$$\begin{aligned} q(x)&=\tfrac{1}{2}x_0 && 0<x<x_0\\ &=\tfrac{1}{2}x && x_0<x<x_2\\ &=\tfrac{1}{2}x_2 && x_2<x<x_1\\ &=\tfrac{1}{2}x_2+\delta q && x_1<x<1. \end{aligned} \tag{11}$$

where we used equation (10). Taking $x\to x_0+0$ in equation (9), we have

$$\bar{q}=\tau+(\tfrac{1}{2}x_0)^2. \tag{12}$$

Taking $x \to x_0+0$ in equation (3), we have

$$(\tfrac{1}{2}x_0)^3 = \tfrac{3}{4}h^2. \tag{13}$$

The δq is determined by substituting $x=x_1+0$ and $x=x_1-0$ in equation (3). For small h_1, we have

$$(\delta q)^2 \simeq h_1^2/(x_1-x_2). \tag{14}$$

The x_2 is determined by using $\int_0^1 q(x)\,dx = \tfrac{1}{4}x_0^2+\tfrac{1}{2}x_2-\tfrac{1}{4}x_2^2+\delta q(1-x_1)$ and equation (12) as follows

$$\tfrac{1}{4}x_2^2-\tfrac{1}{2}x_2+\tau-(1-x_1)h_1/(x_1-x_2)^{1/2}=0. \tag{15}$$

The transition temperature τ_g is determined by taking $x_2 \to x_0$ in this equation. If we consider that the h_1 is infinitesimal, we have the AT line $\tau_g \simeq (\tfrac{3}{4}h^2)^{1/3}$. If we consider that $h=0$ and h_1 is small but finite, we have

$$\tau_{g0}=(1-x_1)h_1/(x_1)^{1/2}. \tag{16}$$

This result shows that if the amplitude of the applied AC field h_1 is finite, the transition temperature T_{g0} decreases linearly to the amplitude h_1, and if the h_1 is constant and the frequency scale x_1 can be changed, T_{g0} decreases linearly to $1-x_1$ at $0 \ll x_1 \leqslant 1$. The phenomenon that T_{g0} decreases as the frequency scale in the measurement decreases has often been observed experimentally (Nire 1982, Lohneysen *et al* 1978).

Next we consider case 2. We have the next solution in this case:

$$\begin{aligned} q(x) &= \tfrac{1}{2}x_0-\delta q && 0<x<x_1 \\ &= \tfrac{1}{2}x_0 && x_1<x<x_0 \\ &= \tfrac{1}{2}x && x_0<x<x_2 \\ &= \tfrac{1}{2}x_2 && x_2<x<1. \end{aligned} \tag{17}$$

From similar considerations to case 1, we have

$$(\delta q)^2 \simeq h_1^2/(x_0-x_1) \tag{18}$$

$$(\tfrac{1}{2}x_0)^3 \simeq \tfrac{3}{4}h^2+\tfrac{3}{4}x_1x_0h_1/(x_0-x_1)^{1/2} \tag{19}$$

$$\tfrac{1}{4}x_2^2-\tfrac{1}{2}x_2+\tau=0 \tag{20}$$

$$\bar{q}=\tau+(\tfrac{1}{2}x_0)^2-x_1\delta q \simeq \tau+(\tfrac{3}{4}h^2)^{2/3}. \tag{21}$$

We notice the difference between the transition temperatures in case 1 and case 2 which are obtained by taking $x_2 \to x_0$ in equations (15) and (20), if there are a small DC field h and a very small finite AC field h_1.

Finally we consider case 3. We have the solution

$$\begin{aligned} q(x) &= \tfrac{1}{2}x_0 && 0<x<x_0 \\ &= \tfrac{1}{2}x && x_0<x<x_1-\delta q \\ &= \tfrac{1}{2}(x_1-\delta q) && x_1-\delta q<x<x_1 \\ &= \tfrac{1}{2}(x_1+\delta q) && x_1<x<x_1+\delta q \\ &= \tfrac{1}{2}x_2 && x_2<x<1. \end{aligned} \tag{22}$$

From similar considerations to case 1, we have

$$\delta q = (3h_1)^{2/3} \tag{23}$$

$$(\tfrac{1}{2}x_0) = (\tfrac{3}{4}h^2)^{1/3} \tag{24}$$

$$\tfrac{1}{4}x_2^2 - \tfrac{1}{2}x_2 + \tau = 0 \tag{25}$$

$$\bar{q} = \tau + (\tfrac{1}{2}x_0)^2. \tag{26}$$

Using equation (7), we have the susceptibilities in the spin glass phase in each case as follows:

case 1

$$\begin{aligned} &G(0) \simeq 1 - (\tfrac{3}{4}h^2)^{2/3} \\ &G(x_1+0) = G(0) \simeq 1 - \tau^2 - x_1 h_1/(x_1 - 2\tau)^{1/2} \end{aligned} \tag{27}$$

case 2

$$\begin{aligned} &G(0) \simeq 1 - (\tfrac{3}{4}h^2)^{2/3} \\ &G(1) \simeq 1 - \tau^2 \\ &G(x_1+0) \simeq 1 - (\tfrac{3}{4}h^2)^{2/3} - x_1 h_1/(2(\tfrac{3}{4}h^2)^{1/3} - x_1)^{1/2} \end{aligned} \tag{28}$$

case 3

$$\begin{aligned} &G(0) \simeq 1 - (\tfrac{3}{4}h^2)^{2/3} \\ &G(1) \simeq 1 - \tau^2 \\ &G(x_1+0) \simeq 1 - \tfrac{1}{4}[x_1 + (3h_1^2)^{1/3}]^2. \end{aligned} \tag{29}$$

The nonlinear susceptibility for the infinitesimal AC field $\chi_2 \equiv \mathrm{d}^2 G(x_1+0)/\mathrm{d}h_1^2|_{h_1=0}$ is divergent only in case 3.

In case 3, if we consider that $h = 0$ and h_1 is small but finite, solution (23) is correct at $x_1 > \delta q$. When $x_1 < \delta q$, we have the next solution

$$q(x) = \begin{cases} 0 & 0 < x < x_1 \\ \delta q = \tfrac{1}{2}x_0 & x_1 < x < x_0 \\ \tfrac{1}{2}x & x_0 < x < x_2 \\ \tfrac{1}{2}x_2 & x_2 < x < 1 \end{cases} \tag{30}$$

where

$$\tfrac{4}{3}(\delta q)^3 - x_1(\delta q)^2 = h_1^2 \tag{31}$$

$$\tfrac{1}{4}x_2^2 - \tfrac{1}{2}x_2 + \tau = 0 \tag{32}$$

$$\bar{q} = \tau + (\delta q)^2 - x_1(\delta q). \tag{33}$$

We define $x_1 \equiv b(\delta q)$ where $0 < b < 1$, we have

$$\delta q = h_1^{2/3}/(\tfrac{4}{3} - b)^{1/3} \tag{34}$$

from equation (31). When $b = 0$, we have the result in the static field (24). When $b = 0$, we have equation (23). The transition temperature τ_g is determined by $x_2 \to x_0$

in equation (32) as

$$\tau_g \simeq h_1^{2/3}/(\tfrac{4}{3}-b)^{1/3}. \tag{35}$$

When $b=0$, the τ_g is identified as the AT line. When b is increased to larger than two, that is, the frequency scale is increased sufficiently, the AT line is shifted to higher field. This phenomenon was observed experimentally (Salamon and Tholence 1983). When $x_1 \gg \delta q$, the transition temperature has a crossover from τ_g by equation (36) to τ_{g0} by equation (16).

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